**MAT 271E Probability and Statistics – 2014-2015 Spring - Final**

Do all parts of all problems. Show your work for credit. Write your name on all submitted sheets. 120 minutes

1. The weight of adult male chimpanzees is distributed as N(50,5) and adult female chimpanzees as N(40,4). Assuming that, on the average, 11 out of 20 chimpanzees are adult males, determine the probability that at least 4 out of 5 chimpanzees randomly selected from the population (male or female) have a weight larger than 45. (20 pts)

2. The joint distribution of two random variables is given as



a) determine the marginal density of random variable X. Specifically identify the density in the intervals i) , ii), iii)  (10 pts)

b) Are the random variables X and Y statistically independent? (5 pts)

c) If r.v. Y assumes value 1.5, what is the probability that random variable X

i) assumes a value larger than 1.5? (5pts)

ii) assumes a value between 0 and 1.25? (5 pts)

3. The cost per litre oil purchased may be modelled according to the following plot



Y:Cost

X:No. Units purchased

a) determine the probability distribution function of random variable Y that represents the cost per litre oil purchased in terms of the probability distribution function of X that represents no. litres of oil purchased. (10 pts)

b) If the no. litres of oil purchased is modelled with the density ,

determine the density of r.v. Y that represents the cost per litre oil purchased. (Hint: Note that there is a discontinuity in  at the discrete realization of r.v. Y.) (5 pts)

4. Consider a Bernoulli random variable that takes on values 0 and 1 with probabilities 1-p and p, respectively. We observe one realization of this random variable, say X and form the estimate  for the unknown probability p .

a) What should the value of the constant k be so that the estimate is unbiased? (10 pts)

b) For what values of k does the above estimate have a smaller mean squared error than the estimate ? You may leave your answer as an inequality in terms of k and p. (10 pts)

5. The variance of the pressure readings with a barometer has been registered as 0.1 (kPascals)2. In order to measure the air pressure on a certain day, a meteorologist takes 16 readings with this barometer. The mean value of these readings is 94.95kPascals.

a) Is it plausible that the true air pressure is greater than or equal to 95kPascals? Use a significance level of 0.05 to test the hypothesis. (15 pts)

b) Is it plausible that the true air pressure is less than 95kPascals? Use a significance level of 0.05 to test the hypothesis. (10 pts)